29 November 2001
PHYSICS LETTERS B
ELSEVIER
Physics Letters B 521 (2001) 320–328
www.elsevier.com/locate/npe

# Complex flavour couplings in supersymmetry and unexpected *CP* violation in the decay $B \to \phi K$

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Received 25 September 2001; accepted 16 October 2001
Editor: P.V. Landshoff

**Abstract**

Complex flavour couplings (off-diagonal mass terms) in the squark sector of supersymmetric theories may drastically alter both the rate and the *CP*-violating asymmetry of certain $B$-meson decays. We consider the effects of couplings that induce $b \to s$ transitions and lead to final state with strangeness one. We investigate the bounds that must be satisfied by the new terms and explore the possible implications on direct and mixing induced asymmetries in the charged and neutral $B \to \mathrm{J}/\Psi K$ and $B \to \phi K$ decays.



## 1. Introduction

Certain decays of $B$-mesons are expected to shed light on the mechanism of *CP* violation and, more generally, on new physics. Indeed, the first surprising results on the *CP* asymmetry $a_{\psi K}$ in the decay $B \to \mathrm{J}/\Psi K$ raised the hope for a first new physics signal. Meanwhile, the value of the measured asymmetry has changed considerably and the present world average [1]

$$a_{\psi K}^{\mathrm{WA}} = 0.79 \pm 0.12 \tag{1}$$

almost coincides with the standard model expectation [2]

$$a_{\psi K}^{\mathrm{SM}} \simeq 0.70 \pm 0.10. \tag{2}$$

Although this came as a disappointment, it reinforced the view that one should be prepared, both on the theoretical and the experimental side, for unexpected observations. In fact, the small error of the above world average and the improvements that are expected during the next years will allow to detect even small deviations from the standard model predictions.

There are of course many new physics scenarios. On the one hand, it is possible to describe their signatures in very general terms (for recent expositions, see, e.g., Refs. [3–5]). On the other hand, one can propose a specific new physics model and investigate its consequences. The latter approach was the topic of a huge body of work. For what concerns supersymmetric models, emphasis was given recently to the so-called minimal flavour violating models (MFV) [6] and their possible variants [7].

In virtually all new physics models, new non-standard fields are introduced and, therefore, new complex (i.e., with *CP*-violating phases) couplings appear. This is well known in supersymmetry to which

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PII: S0370-2693(01)01240-0

we turn for definiteness. In supersymmetric models there are several classes of phases. Those in the $\mu$ and flavour diagonal $A$ terms do not contribute to flavour changing processes but are strongly bound by the electric dipole moment of the neutron and other particles [8]. The phases of the Yukawa couplings are the same as in the standard model. If these are the only phases and flavour changing couplings of a model, the latter belongs to the class of MFV supersymmetric models that exhibit many analogies to the standard model [6]. Therefore, the most interesting phases are those in the squark and slepton mass matrices. Their flavour diagonal elements are either real by definition (in the $LL$ and $RR$ sectors) or small (in the $LR$ and $RL$ ones) as pointed out before (restrictions from electric dipole moments). Therefore the flavour changing elements (i.e., $(m^2)^{u,d}_{23}$ and $(m^2)^{u,d}_{13}$, in the sectors $LL$, $RL$, etc.) are the most interesting ones and we will focus on their effects. They contribute mainly through loop diagrams with internal gluinos and charginos. Since we are interested in contributions to Wilson coefficients that are already quite large in the SM [2] (i.e., the coefficients of the QCD penguin and chromo-magnetic dipole moment operators), we will only discuss gluino loops; in fact, they tend to dominate over the corresponding chargino ones.

Clearly this is not the first analysis of such SUSY diagrams. Their impact on flavour physics is known since a long time [9], and there are numerous recent investigations. To clarify the new aspects provided in this Letter, we review briefly some of the most recent efforts.

In Ref. [10] a first comprehensive analysis of the gluino exchanges was given. The systematics of the perturbative expansion was investigated in Ref. [11], which also included an analysis of the mass insertion approximation: the latter was found to be sufficient in many cases and we will use it again in this study. In Ref. [12] the bounds from the rare decay $b \to s\gamma$ were derived; however, the various couplings were assumed to be real. In Ref. [13], the influence on the electromagnetic penguins (isospin violating terms) and the consequences on $B \to K\pi$ decays were studied; particular focus was given to the determination of the CKM angle $\gamma$. In Ref. [3] effects of new terms on direct *CP*-violating asymmetries in charged decays were given, but the possible values of the new coefficients were not investigated. In Ref. [5] a completely general parameterization of new physics effects in the decays $B \to \phi K$ and $B \to \mathrm{J}/\Psi K$ was given; however, no particular model was explicitly studied. In Ref. [14], the case of left-right symmetric models were investigated and it was found that, in these models, sizeable deviations from the relation $a_{\psi K} = a_{\phi K}$ are possible.

In this Letter, we consider the *complex* $(2,3)$ entries in the down squark mass matrix and work out their consequences for the time dependent *CP* asymmetries in the decays $B \to \phi K$ and $B \to \mathrm{J}/\Psi K$. In the standard model, these asymmetries are equal and are a measure of the phase of the $B_d$–$\overline{B}_d$ mixing amplitude (which in the SM is $2\beta$). The complex couplings that we consider leave the $B_d$–$\overline{B}_d$ mixing phase unchanged but have a significant impact on the amplitudes of the decays. The $B \to \mathrm{J}/\Psi K$ amplitude is dominated by SM tree level contributions and, consequently, the *CP*-violating asymmetry is hardly modified; on the other hand, there is room for such an effect in the transition $B \to \phi K$. This decay is easily accessible at the $B$-factory experiments [15] and an investigation of the related *CP* asymmetries is clearly worthwhile. In the analysis we will concentrate on this decay.

This note is organized as follows. In Section 2 we review the effective Hamiltonian for $b \to s\bar{s}s$ transitions in generic extensions of the standard model and give the explicit expressions of the various *CP* asymmetries. We also discuss the observables $\mathcal{B}(B \to X_s\gamma)$ and $\Delta M_{B_s}$ which are related to the $b \to s$ transition and provide interesting additional pieces of information. In Section 3 we present the SUSY model that we consider and the explicit contributions to the Wilson coefficients. The numerical analysis is presented in Section 4. The impact of the complex SUSY parameters on the branching ratio and *CP* asymmetry of $B \to X_s\gamma$ is explored and we analyze the correlation between the mixing-induced (and direct) *CP* asymmetries in $B \to \phi K$ and the $B_s$–$\overline{B}_s$ mass difference. A brief summary of our results and some comments are found in Section 5.

[2] Contributions to the small electroweak penguin coefficients are usually negligible because they do not appreciably change the standard model predictions.

## 2. Effective Hamiltonian

The effective Hamiltonian for $b \to s$ transitions can be written as

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \Bigg[ \sum_{i=1}^{10} C_i(\mu) \mathcal{O}_i + C_{7\gamma} \mathcal{O}_{7\gamma} + C_{8g} \mathcal{O}_{8g} \Bigg], \tag{3}$$

where ($e_{u,c} = 2/3$ and $e_{d,s,b} = -1/3$)

$$\mathcal{O}_1 = (\bar{c}_\alpha b_\beta)_{V-A} (\bar{s}_\beta c_\alpha)_{V-A}, \tag{4}$$

$$\mathcal{O}_2 = (\bar{c} b)_{V-A} (\bar{s} c)_{V-A}, \tag{5}$$

$$\mathcal{O}_3 = (\bar{s} b)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q} q)_{V-A}, \tag{6}$$

$$\mathcal{O}_4 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V-A}, \tag{7}$$

$$\mathcal{O}_5 = (\bar{s} b)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q} q)_{V+A}, \tag{8}$$

$$\mathcal{O}_6 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V+A}, \tag{9}$$

$$\mathcal{O}_7 = \frac{3}{2} (\bar{s} b)_{V-A} \sum_{q=u,d,s,c,b} e_q (\bar{q} q)_{V-A}, \tag{10}$$

$$\mathcal{O}_8 = \frac{3}{2} (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s,c,b} e_q (\bar{q}_\beta q_\alpha)_{V-A}, \tag{11}$$

$$\mathcal{O}_9 = \frac{3}{2} (\bar{s} b)_{V-A} \sum_{q=u,d,s,c,b} e_q (\bar{q} q)_{V+A}, \tag{12}$$

$$\mathcal{O}_{10} = \frac{3}{2} (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s,c,b} e_q (\bar{q}_\beta q_\alpha)_{V+A}, \tag{13}$$

$$\mathcal{O}_{7\gamma} = \frac{e}{4\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, \tag{14}$$

$$\mathcal{O}_{8g} = \frac{g_s}{4\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}. \tag{15}$$

In Eq. (3), we did not write the operators $\mathcal{O}^u_{1,2}$ obtained by the replacements $c \to u$ in $\mathcal{O}_{1,2}$, and the semileptonic current–current operators which induce the transitions $b \to s\ell^+\ell^-$. The effective Hamiltonian for $b \to d$ transitions can be obtained via the substitution $s \to d$. Also, operators with different helicity structures are not explicitly written and can be obtained from the above operator basis via the replacement $L \leftrightarrow R$ [11]. We will comment on their impact on the numerical analysis in Section 4.

Table 1
SM Wilson coefficients at the scale $m_b$

| | | | |
|---|---|---|---|
| $C_1$ | $-0.171$ | $C_7$ | $-0.00001$ |
| $C_2$ | $1.070$ | $C_8$ | $0.0005$ |
| $C_3$ | $0.0114$ | $C_9$ | $-0.01$ |
| $C_4$ | $-0.0321$ | $C_{10}$ | $0.0002$ |
| $C_5$ | $0.00925$ | $C_{7\gamma}$ | $-0.313$ |
| $C_6$ | $-0.0383$ | $C_{8g}$ | $-0.188$ |

The SM values of Wilson coefficients $C_{1-10}(m_b)$ and $C_{7\gamma,8g}(m_b)$ are given in Table 1; the opposite chirality operators do not get standard model contributions if the light masses are neglected.

We will use the following approximate form[3] for the $B \to (\mathrm{J}/\Psi, \phi)K$ amplitudes [3]:

$$\mathcal{A}^\psi = -\sqrt{2} G_F f_\psi F_1^{B\to K}\big(m_\psi^2\big) m_\psi (\epsilon_\psi \cdot p_B) \times V_{tb} V_{ts}^* \bigg[ C_\psi + \frac{\alpha_s}{2\pi} \frac{m_B^2}{m_\psi^2} C_{8g} r_8 \widetilde{S}_{\psi K} \bigg], \tag{16}$$

$$\mathcal{A}^\phi = -\sqrt{2} G_F f_\phi F_1^{B\to K}\big(m_\phi^2\big) m_\phi (\epsilon_\phi \cdot p_B) \times V_{tb} V_{ts}^* \bigg[ C_\phi + \frac{8}{9} P + \frac{\alpha_s}{4\pi} \frac{m_b^2}{q^2} C_{8g} \widetilde{S}_{\phi K} \bigg]. \tag{17}$$

Here, $f_{\psi(\phi)}$ is the decay constant of the $\mathrm{J}/\Psi(\phi)$, $F_1^{B\to K}(q^2)$ is the $B \to K$ penguin form factor, $\widetilde{S}_{(\phi,\psi)K} \simeq -0.76$ is the ratio of the chromo-magnetic to penguin form factors for the $B \to (\mathrm{J}/\Psi, \phi)K$ decay, $q^2 \simeq m_b^2/2$, and $r_8 \simeq 1/12$ is the ratio of colour octet and singlet matrix elements. $P$ is an $O(\alpha_s)$ contribution to the matrix elements of the QCD penguin operators, it is insensitive to new short distance physics, and it is originated by loop diagrams with an internal charm quark. Since the typical $q^2$ is above the charm production threshold, $P$ carries a strong phase and its numerical value is $-0.0132 - i\,0.0145$ [16]. Finally, the coefficients $C_{\psi,\phi}$ are

$$C_\psi = C_1 + C_3 + C_5 + \frac{C_2 + C_4 + C_6}{3} + 2r_8(C_2 + C_4 + C_6), \tag{18}$$

[3] This expression is based on naive factorization and is therefore not exact; but it correctly describes the way new physics enters.

$$C_\phi = C_3 + C_4 + C_5 + \frac{C_3 + C_4 + C_6}{3} - \frac{1}{2}\left(C_7 + C_9 + C_{10} + \frac{C_8 + C_9 + C_{10}}{3}\right), \tag{19}$$

where all the $C_i$ are to be evaluated at the scale $m_b$.

New physics contribute to the above Wilson coefficients; the resulting new phases will show up in the direct and mixing induced *CP* asymmetries. Before writing an explicit model for such contributions, we briefly discuss the renormalization group (RG) running of $C_{\psi,\phi}(\mu)$ from $\mu = O(M_W)$ to $\mu = O(m_b)$. The solution of the NLO RG equations reads:

$$C_\psi(m_b) = 0.3515\big(1 + 0.0018\, R_3 - 0.0014\, R_4 + 0.0020\, R_5 + 0.0053\, R_6\big), \tag{20}$$

$$C_\phi(m_b) = -0.0243\big(1 - 0.030\, R_3 + 0.085\, R_4 - 0.028\, R_5 + 0.044\, R_6\big), \tag{21}$$

where $R_i \equiv C_i(M_W)/C_i^{\mathrm{SM}}(M_W)$ and the other coefficients are fixed to their SM value. It follows that the impact of the QCD penguin matching conditions (the values of the Wilson coefficients at the scale $M_W$) on $C_\psi(m_b)$ and $C_\phi(m_b)$ is respectively of order 1% and 10%. Thus, as it is well known, their effects on $C_\psi(m_b)$ is totally negligible: the coefficients of the current–current operators totally dominate the amplitude even if their contribution is colour suppressed. Thus, the SM prediction of an extremely small direct *CP* asymmetry in the $\mathrm{J}/\Psi$ mode is unaltered by new physics contributions. On the other hand, new physics whose contributions are large enough to dominate the phases of $C_{3-6}(M_W)$, may modify the overall phase of $C_\phi(m_b)$ at the 10% level. Taking into account that the *CP* asymmetry depends on twice the phase of $C_\phi(m_b)$, we see that in principle large ($O(0.2)$) deviations from the SM relation $a_{\psi K} = a_{\phi K}$ are possible. Moreover, the presence of a term proportional to $C_{8g}$ in Eq. (17) can have a strong impact if there are large new complex contributions to the chromo-magnetic dipole operator. In the standard model, this term is negligible.

Let us introduce the time-dependent *CP* asymmetry in the decays $B_d^0/\overline{B}_d^0 \to \phi K$, given by

$$a_{\phi K}(t) \equiv \frac{\Gamma\big(B_d^0(t) \to \phi K\big) - \Gamma\big(\overline{B}_d^0(t) \to \phi K\big)}{\Gamma\big(B_d^0(t) \to \phi K\big) + \Gamma\big(\overline{B}_d^0(t) \to \phi K\big)} = \mathcal{A}_{CP}^{\mathrm{dir}} \cos\big(\Delta M_{B_d} t\big) + \mathcal{A}_{CP}^{\mathrm{mix}} \sin\big(\Delta M_{B_d} t\big), \tag{22}$$

where $\Delta M_{B_d}$ is the is the $B_d$–$\overline{B}_d$ mass difference and $B_d^0(t)$ ($\overline{B}_d^0(t)$) is the state at time $t$ which started as a pure $B_d^0$ ($\overline{B}_d^0$) at $t = 0$. $\mathcal{A}_{CP}^{\mathrm{dir}}$ and $\mathcal{A}_{CP}^{\mathrm{mix}}$ are the direct and mixing-induced *CP*-asymmetries, respectively. Their explicit expressions are

$$\mathcal{A}_{CP}^{\mathrm{dir}} = \frac{1 - |\lambda_{\phi K}|^2}{1 + |\lambda_{\phi K}|^2}, \tag{23}$$

$$\mathcal{A}_{CP}^{\mathrm{mix}} = \frac{2\,\mathrm{Im}\,\lambda_{\phi K}}{1 + |\lambda_{\phi K}|^2}, \tag{24}$$

with

$$\lambda_{\phi K_S} = e^{2i(\beta+\theta_d)} \frac{\bar{A}}{A} \equiv e^{2i(\beta+\theta_d+\theta_A)} \left|\frac{\bar{A}}{A}\right|. \tag{25}$$

Here, $\beta$ is the inner angle of the unitarity triangle of the standard model, $\theta_d$ is a possible new physics contribution to the phase of the $B_d$–$\overline{B}_d$ oscillations, and $A$ ($\bar{A}$) are the amplitudes of the decay in question (and its *CP*-conjugate).

We turn next to the branching ratio and *CP* asymmetry in the decay $B \to X_s\gamma$ and to the $B_s$–$\overline{B}_s$ mass difference. These observables are strongly affected by new physics in the FCNC $b \to s$ transition and therefore must be properly accounted for.

### *2.1. $B \to X_s\gamma$*

The inclusive transition $B \to X_s\gamma$ plays a major role in limiting possible new physics contributions to $B$ decays both through constraints on the branching ratio and on direct *CP* asymmetry. The experimental information on the latter quantities is [17]

$$\mathcal{B}(B \to X_s\gamma) = (3.22 \pm 0.40) \times 10^{-4}, \tag{26}$$

$$\mathcal{A}_{CP}(B \to X_s\gamma) = (-3.5 \pm 7.7)\%. \tag{27}$$

In the numerical analysis we will use the NLO computation of $\mathcal{B}(B \to X_s\gamma)$ which, as a function of the Wilson coefficients evaluated at the scale $M_W$ reads [18]:

$$\mathcal{B} = \big[1.258 + 0.382\,|R_7|^2 + 0.015\,|R_8|^2 + 1.395\,\mathrm{Re}\,R_7 + 0.161\,\mathrm{Re}\,R_8 + 0.083\,\mathrm{Re}\big(R_7 R_8^*\big)\big] \times 10^{-4}, \tag{28}$$

where $R_{7,8} = C_{7\gamma,8g}^{\mathrm{tot}}(M_W)/C_{7\gamma,8g}^{\mathrm{SM}}(M_W)$. The parameter $\delta$ that determines the cut on the photon energy spectrum is set to 0.90 according to Ref. [18].

Similarly, the $CP$ can be written as [19]:

$$\mathcal{A}_{CP} = 1.06\,\mathrm{Im}\,\frac{C_2(m_b)}{C_{7\gamma}(m_b)} - 9.52\,\mathrm{Im}\,\frac{C_{8g}(m_b)}{C_{7\gamma}(m_b)} + 0.16\,\mathrm{Im}\,\frac{C_2(m_b)C_{8g}^*(m_b)}{|C_{7\gamma}(m_b)|^2}. \tag{29}$$

*2.2.* $\Delta M_{B_s}$

The effective Hamiltonian for the $\Delta S = 2$ transitions can be written as

$$\mathcal{H}_{\mathrm{eff}}^{\Delta S=2} = -\frac{G_F^2 M_W^2}{(2\pi)^2}\big(V_{tb}V_{ts}^*\big)^2\bigg[C^{VLL}\mathcal{O}^{VLL} + \sum_{i=1}^{2}\big(C_i^{SLL}\mathcal{O}_i^{SLL} + C_i^{SLR}\mathcal{O}_i^{SLR}\big)\bigg] + \text{h.c.}, \tag{30}$$

where

$$\mathcal{O}^{VLL} = \big(\bar{s}_L\gamma^\mu b_L\big)\big(\bar{s}_L\gamma_\mu b_L\big), \tag{31}$$

$$\mathcal{O}_1^{SLL} = \big(\bar{s}_L b_R\big)\big(\bar{s}_L b_R\big), \tag{32}$$

$$\mathcal{O}_2^{SLL} = \big(\bar{s}_L^\alpha b_R^\beta\big)\big(\bar{s}_L^\beta b_R^\alpha\big), \tag{33}$$

$$\mathcal{O}_1^{SLR} = \big(\bar{s}_L b_R\big)\big(\bar{s}_R b_L\big), \tag{34}$$

$$\mathcal{O}_2^{SLR} = \big(\bar{s}_L^\alpha b_R^\beta\big)\big(\bar{s}_R^\beta b_L^\alpha\big), \tag{35}$$

together with the operators $\mathcal{O}^{VRR}$ and $\mathcal{O}_{1,2}^{SRR}$, obtained from the corresponding $LL$ ones via the substitution $L \leftrightarrow R$.

In order to minimize the impact of hadronic uncertainties we will consider, as usual, the ratio $X_{sd} \equiv \Delta M_{B_s}/\Delta M_{B_d}$. Its explicit expression is [20]:

$$X_{sd} = \xi^2 \frac{M_{B_s}}{M_{B_d}}\left|\frac{V_{ts}}{V_{td}}\right|^2 \frac{\mathcal{C}^s}{\mathcal{C}^d}, \tag{36}$$

where

$$\mathcal{C}^s = C^{VLL} - \frac{5\kappa_s}{8}C_1^{SLL} + \frac{\kappa_s}{8}C_2^{SLL} + \frac{6\kappa_s+1}{8}C_1^{SLR} + \frac{2\kappa_s+3}{8}C_2^{SLR} \tag{37}$$

with $\kappa_s = M_{B_s}^2/(m_b+m_s)^2$ and $\xi$ is a ratio of hadronic matrix elements, numerically equal to $1.16 \pm 0.05$ [2]. The coefficient $\mathcal{C}^d$ is obtained by replacing $s$ with $d$ in Eq. (37). It is important to stress that the contributions from new physics are generally different for the $B_s$ and $B_d$ systems. The ratio $\mathcal{C}^s/\mathcal{C}^d$ can therefore be sizeably different from unity and has to be taken into account in model independent analyses of the unitary triangle when using Eq. (36).

In the numerical analysis we will require $X_{sd}$ to lay in the interval (30–60) in order to satisfy the lower bound $\Delta M_{B_s} > 14.9\ \mathrm{ps}^{-1}$ and have an observable $B_s$–$\overline{B}_s$ mass difference at the same time.

## 3. An example: SUSY gluino contributions

In order to present a definite new physics model which contributes sizeably to the $CP$ asymmetries in $B \to \phi K$, we turn to a variant of the MSSM with complex off-diagonal squark mass terms. In particular, we will focus on the following entries: $(m_{LL}^2)_{23}^d$, $(m_{LR}^2)_{23}^d$, $(m_{RR}^2)_{23}^d$ and $(m_{RL}^2)_{23}^d$. The other parameters of the model are the common mass of the squarks $\widetilde{m}$, and the gluino mass $m_{\tilde{g}}$. Following a common practice, we consider the normalized insertions $\delta_{23}^d$ given by the ratios of the various $(m^2)_{23}^d$ to $\widetilde{m}^2$.

It is well known, and in the next section we will give a detailed quantitative analysis of the question, that $\mathcal{B}(B \to X_s\gamma)$ puts severe bounds of order $O(10^{-2})$ on the $LR$ and $RL$ insertions while the impact on the $LL$ and $RR$ ones is rather mild. This strong result follows from the $m_{\tilde{g}}/m_b$ chiral enhancement of the $(\delta_{23}^d)_{LR,RL}$ contributions to the Wilson coefficient $C_{7\gamma,8g}$. Note that this chiral factor is absent in the in the WC's that govern the $B_s$–$\overline{B}_s$ mass difference and the $B \to \phi K$ amplitude. For the latter observables, in fact, all the mass insertions enter with similar weight: therefore, the only insertions that can play a role are the $LL$ and $RR$ ones. Since the analyses for both insertions give the same results, we will consider explicitly only $(\delta_{23}^d)_{LL}$.

The expressions of the SUSY contributions to the various coefficients are [21]

$$C_3 = \frac{\alpha_s^2(\delta_{23}^d)_{LL}}{2\sqrt{2}\,G_F\widetilde{m}^2|V_{tb}V_{ts}^*|} \times \bigg[-\frac{1}{9}B_1(x_{\tilde{g}\tilde{q}}) - \frac{5}{9}B_2(x_{\tilde{g}\tilde{q}}) - \frac{1}{3}P(x_{\tilde{g}\tilde{q}})\bigg], \tag{38}$$

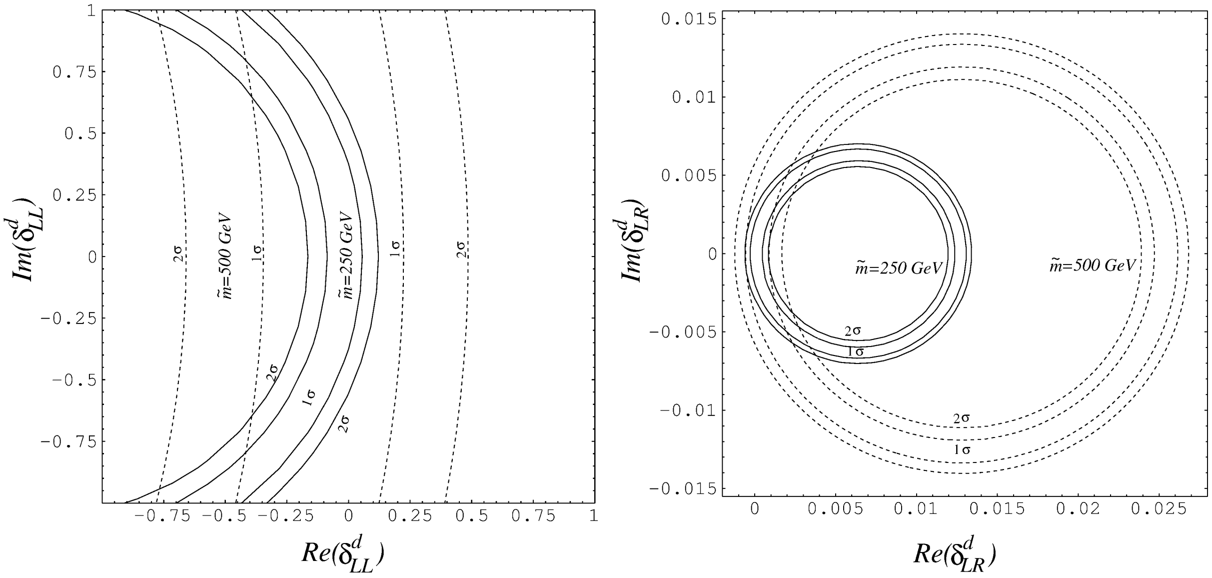


Fig. 1. Constraints on the complex mass insertions $(\delta^d_{23})_{LL}$ and $(\delta^d_{23})_{LR}$ coming from $\mathcal{B}(B \to X_s\gamma)$. We consider $m^2_{\tilde{g}}/\widetilde{m}^2 = 1$. The contours scale as $\widetilde{m}^2$. In the plot we explicitly show the cases $\widetilde{m} = 250, 500$ GeV.

$$C_4 = \frac{\alpha_s^2(\delta^d_{23})_{LL}}{2\sqrt{2}\,G_F\widetilde{m}^2|V_{tb}V^*_{ts}|} \times \left[-\frac{7}{3}B_1(x_{\tilde{g}\tilde{q}}) + \frac{1}{3}B_2(x_{\tilde{g}\tilde{q}}) + P(x_{\tilde{g}\tilde{q}})\right], \tag{39}$$

$$C_5 = \frac{\alpha_s^2(\delta^d_{23})_{LL}}{2\sqrt{2}\,G_F\widetilde{m}^2|V_{tb}V^*_{ts}|} \times \left[\frac{10}{9}B_1(x_{\tilde{g}\tilde{q}}) + \frac{1}{18}B_2(x_{\tilde{g}\tilde{q}}) - \frac{1}{3}P(x_{\tilde{g}\tilde{q}})\right], \tag{40}$$

$$C_6 = \frac{\alpha_s^2(\delta^d_{23})_{LL}}{2\sqrt{2}\,G_F\widetilde{m}^2|V_{tb}V^*_{ts}|} \times \left[-\frac{2}{3}B_1(x_{\tilde{g}\tilde{q}}) + \frac{7}{18}B_2(x_{\tilde{g}\tilde{q}}) + P(x_{\tilde{g}\tilde{q}})\right], \tag{41}$$

$$C_{7\gamma} = -\frac{\pi\alpha_s(\delta^d_{23})_{LL}}{\sqrt{2}\,G_F\widetilde{m}^2|V_{tb}V^*_{ts}|}\frac{16}{9}g_2(x_{\tilde{g}\tilde{q}}), \tag{42}$$

$$C_{8g} = -\frac{\pi\alpha_s(\delta^d_{23})_{LL}}{\sqrt{2}\,G_F\widetilde{m}^2|V_{tb}V^*_{ts}|} \times \left(\frac{1}{3}g_2(x_{\tilde{g}\tilde{q}}) + 3g_1(x_{\tilde{g}\tilde{q}})\right), \tag{43}$$

$$C^{VLL} = \frac{\alpha_s(\delta^d_{23})_{LL}}{\widetilde{m}^2|V_{tb}V^*_{ts}|^2} \times \left(\frac{1}{9}x_{\tilde{g}\tilde{q}}f_6(x_{\tilde{g}\tilde{q}}) + \frac{23}{72}\tilde{f}_6(x_{\tilde{g}\tilde{q}})\right), \tag{44}$$

where $x_{\tilde{g}\tilde{q}} \equiv m^2_{\tilde{g}}/m^2_{\tilde{q}}$ (we assume a common squark mass). The loop functions can be found in Refs. [20, 21]. Note that we have included the phase of $V_{tb}V^*_{ts}$ (which is basically a minus sign) in the definition of the mass insertion.

## 4. Numerical analysis

We begin with the constraints imposed by the $B \to X_s\gamma$ branching ratio and *CP* asymmetry. We assume the absence of so-called accidental cancellations between different large contributions; therefore, we present the analysis assuming the presence of only one insertion per time besides the SM. If all of the insertions are substantial at the same time, the allowed ranges of some combination of them can be sizeably enlarged (see, for instance, Ref. [12], where the case of multiple real mass insertions is considered). In Fig. 1 we take $x_{\tilde{g}\tilde{q}} = 1$, $\widetilde{m} = 250$, 500 GeV

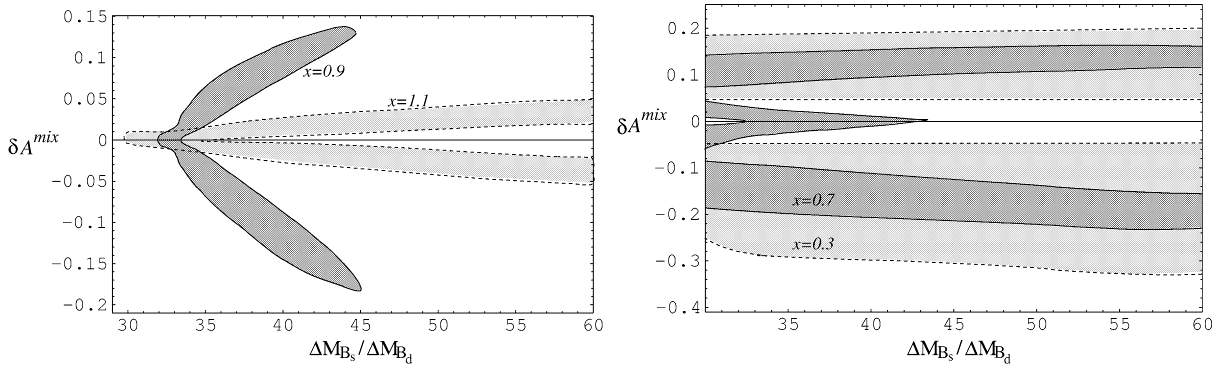


Fig. 2. Correlation between the $B_s$–$\overline{B}_s$ mass difference and the possible deviations $\delta A^{\text{mix}}$ of the mixing-induced *CP* asymmetry from the SM expectation. The various regions correspond to different values of $x_{\tilde{g}\tilde{q}}$.

and require $|\delta| < 1$; we also show the impact of using the 68% C.L. and 95% C.L. constraints on the $B \to X_s\gamma$ branching ratio. The corresponding bounds induced by the *CP* asymmetry are much weaker and do not impact, at the moment, the allowed areas: future experimental improvements can substantially modify this picture. We plot the resulting regions in the $[\mathrm{Re}\,\delta, \mathrm{Im}\,\delta]$ plane. Note that the contours scale with $\widetilde{m}^2$. As mentioned before, the bounds on the $LL$ insertion are not very strong. Even for very light squarks (i.e., $\widetilde{m} = 250$ GeV) it is possible to largely evade the constraint if the imaginary part of the insertion is $O(1)$. This reflects the absence of interference with the (real) SM contribution. On the other hand, the corresponding bounds on the $LR$ insertion are, as expected, of order $O(10^{-2})$; moreover, as it follows from the figure, if $|\delta_{LR}|$ is larger than $O(10^{-3})$, a substantial correlation between real and imaginary parts is required. For the $\delta^d_{RR}$ and $\delta^d_{RL}$ insertions the results are similar.

Next, we analyze the possible deviations of the *CP* asymmetries in $B \to \phi K$ from the SM expectation and the relation to the $B_s$–$\overline{B}_s$ mass difference which is also altered by the insertions. In Fig. 2 we plot the correlation between the $B_s$–$\overline{B}_s$ mass difference and $\delta A^{\text{mix}} \equiv A^{\text{mix}} - A^{\text{mix,SM}}$. We scan over the input SUSY parameters in the ranges

$$\widetilde{m} \in [250, 1000]\ \text{GeV}, \tag{45}$$

$$\left|\left(\delta^d_{23}\right)_{LL}\right| \in [0, 1], \tag{46}$$

$$\arg\left(\delta^d_{23}\right)_{LL} \in [0, 2\pi], \tag{47}$$

for different values of $x_{\tilde{g}\tilde{q}}$, the gluino–squark mass ratio. We require each point to satisfy the $B \to X_s\gamma$ 95% C.L. constraint and to give a $B_s$–$\overline{B}_s$ mass difference in the range $30 \leqslant X_{sd} \leqslant 60$. The various regions correspond to the limiting case: for a given ratio $x_{\tilde{g}\tilde{q}}$, no point lays outside them. For $x_{\tilde{g}\tilde{q}} \geqslant 1$ we find that the deviations from the SM expectation remain below 0.05. For smaller values of $x_{\tilde{g}\tilde{q}}$ much larger and thus observable contributions are possible. This strong dependence on the ratio between the gluino and squark masses is due to the particular dependence of the loop functions on $x_{\tilde{g}\tilde{q}}$. Moreover, the presence of definite bands in the $[X_{sd}, \delta A^{\text{mix}}]$ plane is due to an interplay between the $B \to X_s\gamma$ constraint and the requirement of fixed $\Delta M_{B_s}$.

As previously stated, $\delta A^{\text{mix}}$ measures the deviation of the mixing-induced *CP* asymmetry from the standard model prediction. Since all new physics in the $B_d$–$\overline{B}_d$ mixing amplitude will affect the *CP* asymmetries in the decays $B \to \mathrm{J}/\Psi K$ and $B \to \phi K$ in the same way, $\delta A^{\text{mix}}$ is equal to $\sin 2(\beta + \theta_d + \theta_A) - \sin 2(\beta + \theta_d)$ where $\theta_d$ can be generated by other SUSY couplings that we do not consider here and $\theta_A$ is the phase of the $B \to \phi K$ decay amplitude. In other words, $\delta A^{\text{mix}}$ is the difference between the mixing-induced *CP* asymmetries in the decays $B \to \mathrm{J}/\Psi K$ and $B \to \phi K$ (see also the discussion in Section 2).

The new weak phases in the $B \to \phi K$ amplitude also leads to a non vanishing direct *CP* asymmetry, $A^{\text{dir}}$, which can be measured for instance in decays of charged $B$-mesons. This asymmetry depends crucially on the presence of a strong rescattering phase, pro-

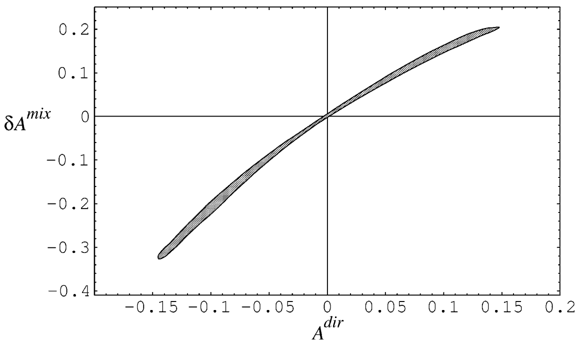


Fig. 3. Correlation between $\delta A^{\mathrm{mix}}$ and the direct *CP* asymmetry $A^{\mathrm{dir}}$.

vided by the term $P$ of Eq. (17). Nevertheless, we find that the new physics contributions to the two asymmetries are strongly correlated. This can be understood as follows. Let us parametrize the $B \to \phi K$ decay amplitude as $A_1 e^{i\phi_1} e^{i\delta_1} + A_2 e^{i\phi_2} e^{i\delta_2}$ where $\phi_i$ and $\delta_i$ are weak and strong phases, respectively; note that this parametrization is arbitrary but that all physical observables do not depend on its choice. In the SUSY model that we consider, the weak phases $\phi_i$ are entirely due to the imaginary part of the mass insertions. Using the above parametrization, we find that the ratio $A^{\mathrm{dir}}/\delta A^{\mathrm{mix}}$ has an extremely tiny dependence on the phases $\phi_i$; therefore, the correspondence between $A^{\mathrm{dir}}$ and $\delta A^{\mathrm{mix}}$ is almost one-to-one. In Fig. 3 we explicitly show this correlation.

## 5. Conclusions

We considered the effects of sizeable flavour changing entries in the squark matrices on $b \to s$ transitions. In particular we allowed for complex values of the relevant off-diagonal elements.

We first investigated the bounds that these entries must obey in order to satisfy the $b \to s\gamma$ data. Fig. 1 shows that the inclusion of complex values for the mass insertion parameters strongly enlarges their allowed regions. In fact, the absolute values of the insertions can be much larger than in the existing literature where only real couplings were considered. Moreover, there are interesting correlations between real and imaginary parts which may give important hints on the structure of the underlying theory.

We then considered the influence of the new terms on the *CP*-violating asymmetry in the decays $B \to \phi K$ and on the $B_s$–$\overline{B}_s$ mixing (the $B_d$–$\overline{B}_d$ mixing is not affected by the terms we are interested in). In Fig. 2, we plot the deviation $\delta A^{\mathrm{mix}}$ of the mixing-induced *CP* asymmetry in $B \to \phi K$ from the SM expectation versus the ratio $X_{sd}$. We see that $\delta A^{\mathrm{mix}}$ can reach the 20% level in some corners of the parameter space; on the other hand, deviations of order $O(10\%)$ are easily possible with moderately light squark and gluino masses. Note that such large contributions are possible only for configurations in which $m_{\tilde{g}} \lesssim m_{\tilde{q}}$. In a similar fashion, Fig. 3 shows that the direct *CP* asymmetry $A^{\mathrm{dir}}$ can receive contributions of the same order of magnitude. We stress that in this framework effects on $B \to J/\Psi K$ decays are expected to be tiny.

The $B_s$–$\overline{B}_s$ $\Delta M_{B_s}$ is very sensitive to the mass insertions we consider, while $\Delta M_{B_d}$ remains unaffected. This implies that the determination of $V_{td}$ from ratio $\Delta M_{B_s}/\Delta M_{B_d}$ may be misleading. Moreover, as it follows from Figs. 2 and 3, an experimental determination of $\Delta M_{B_s}$ in excess with respect to the SM prediction together with sizeable $\delta A^{\mathrm{mix}}$ and $A^{\mathrm{dir}}$ would be strong signatures in favour of this kind of models. During the next year, the $B$-factories BABAR and BELLE will gather enough luminosity to study the *CP* asymmetries in $B \to \phi K$ decays and will test this class of SUSY models soon.

## Acknowledgements

E.L. acknowledges financial support from the Alexander Von Humboldt Foundation. D.W. is partially supported by Schweizerischer Nationalfonds.

## References

[1] K. Ackerstaff et al., OPAL Collaboration, Eur. Phys. J. C 5 (1998) 379;
T. Affolder et al., CDF Collaboration, Phys. Rev. D 61 (2000) 072005;
C.A. Blocker, CDF Collaboration, Proceedings of 3rd Workshop on Physics and Detectors for DAPHNE (DAPHNE 99), Frascati, Italy, 16–19 November 1999, in press;
R. Barate et al., ALEPH Collaboration, Phys. Lett. B 492 (2000) 259;
B. Aubert et al., BABAR Collaboration, Phys. Rev. Lett. 87 (2001) 091801;

K. Abe et al., BELLE collaboration, Phys. Rev. Lett. 87 (2001) 091802.
[2] S. Mele, Phys. Rev. D 59 (1999) 113011;
S. Plaszczynski, M.-H. Schune, hep-ph/9911280;
M. Bargiotti et al., Riv. Nuovo Cimento 23 (3) (2000) 1;
A. Ali, D. London, Eur. Phys. J. C 18 (2001) 665;
M. Ciuchini et al., JHEP 0107 (2001) 013;
A.J. Buras, hep-ph/0101336;
D. Atwood, A. Soni, Phys. Lett. B 508 (2001) 17;
A. Hocker, H. Lacker, S. Laplace, F.L. Diberder, Eur. Phys. J. C 21 (2001) 225.
[3] W.S. Hou, 4th International Workshop on Particle Physics Phenomenology, Kaohsiung, Taiwan, China, 18–21 June 1998, and Workshop on CP Violation, Adelaide, Australia, 3–8 July 1998. Published in “Adelaide 1998, CP Violation”, pp. 13–22, hep-ph/9902382.
[4] Y. Nir, Lectures given at 27th SLAC Summer Institute on Particle Physics: CP Violation in and Beyond the Standard Model (SSI 99), Stanford, California, 7–16 July 1999, hep-ph/9911321;
Y. Grossmann, M. Worah, Phys. Lett. B 395 (1997) 241.
[5] R. Fleischer, T. Mannel, Phys. Lett. B 506 (2001) 311;
R. Fleischer, T. Mannel, Phys. Lett. B 511 (2001) 240.
[6] M. Ciuchini, G. Degrassi, P. Gambino, G.F. Giudice, Nucl. Phys. B 534 (1998) 3;
A. Ali, D. London, Eur. Phys. J. C 9 (1999) 687;
A. Ali, D. London, Phys. Rep. 320 (1999) 79;
A.J. Buras et al., Phys. Lett. B 500 (2001) 161;
A.J. Buras, R. Buras, Phys. Lett. B 501 (2001) 223;
A. Bartl et al., hep-ph/0103324;
A.J. Buras, R. Fleischer, hep-ph/0104238.
[7] A. Ali, E. Lunghi, hep-ph/0105200;
A.J. Buras, P.H. Chankowski, J. Rosiek, L. Slawianowska, hep-ph/0107048.
[8] V. Barger et al., Phys. Rev. D 64 (2001) 056007.
[9] J. Donoghue, H.P. Nilles, D. Wyler, Phys. Lett. B 128 (1983) 55.
[10] A. Masiero, F. Borzumati, S. Bertolini, G. Ridolfi, Nucl. Phys. B 353 (1991) 591.
[11] F. Borzumati et al., Phys. Rev. D 62 (2001) 075005.
[12] T. Besmer, C. Greub, T. Hurth, Nucl. Phys. B 609 (2001) 359.
[13] Y. Grossman, M. Neubert, A. Kagan, JHEP 9910 (1999) 029.
[14] G. Barenboim, J. Bernabeu, M. Raidal, Phys. Rev. Lett. 80 (1998) 4625.
[15] B. Aubert et al., BABAR Collaboration, hep-ex/0105001.
[16] N.G. Deshpande, X.G. He, Phys. Lett. B 336 (1994) 471.
[17] R. Barate et al., ALEPH Collaboration, Phys. Lett. B 429 (1998) 169;
K. Abe et al., BELLE Collaboration, Phys. Lett. B 511 (2001) 151;
D. Cassel, CLEO Collaboration, Talk presented at the XX International Symposium on Lepton and Photon Interactions at High Energies, Rome, Italy, July 23–28, 2001. To be published in the Proceedings;
J. Nash, BABAR Collaboration, Talk presented at the XX International Symposium on Lepton and Photon Interactions at High Energies, Rome, Italy, July 23–28, 2001. To be published in the Proceedings.
[18] A. Kagan, M. Neubert, Eur. Phys. J. C 7 (1999) 5.
[19] A. Kagan, M. Neubert, Phys. Rev. D 58 (1998) 094012.
[20] F. Gabbiani, E. Gabrielli, A. Masiero, L. Silvestrini, Nucl. Phys. B 477 (1996) 321.
[21] A. Ahrib, C.K. Chua, W.S. Hou, hep-ph/0104122.